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A NEW METHOD OF CORRELATING HEAT-TRANSFER COEFFICIENTS FOR NATURAL CONVECTION IN HORIZONTAL CYLINDRICAL ANNULI

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NOMENCLATURE

D_1, outer diameter of inner cylinder of annulus;
D_2, inner diameter of outer cylinder of annulus;
Gr, Grashof number;
Gr_m, Grashof number based on $r_m \, . \ln(r_2/r_1)$;
g, acceleration due to gravity;
h_1, heat-transfer coefficient at r_1;
h_2, heat-transfer coefficient at r_2;
k, thermal conductivity;
k_c, effective conductivity of heat;
l, wetted perimeter of annulus;
Nu, Nusselt number;
$\overline{Nu}$, mean Nusselt number;
Pr, Prandtl number;
Ra, Rayleigh number;
r_1, outer radius of inner cylinder of annulus;
r_2, inner radius of outer cylinder of annulus;
Q_{cond}, heat-transfer rate by conduction only;
Q_{conv}, heat-transfer rate by convection;
S, sectional area of annulus.

Greek symbols
β, coefficient of expansion;
δ, gap width $= r_2 - r_1$;
θ_1, higher temperature at r_1;
θ_2, lower temperature at r_2;
θ_m, mean temperature $= (\theta_1 + \theta_2)/2$;
ν, kinematic viscosity.

Subscripts
1, inner cylinder;
2, outer cylinder.

INTRODUCTION

THERE have been experimental investigations [1–10] on natural convection heat transfer in horizontal cylindrical annuli. Furthermore, several numerical solutions for this problem have been obtained recently [10–12] using large digital computers. However it is still impossible to solve the problem analytically because boundary-layer approximation cannot be applied. So a method of correlating heat-transfer coefficients has not yet been established, and various correlations have been proposed by many authors.

C. Y. Liu *et al.* [3] used k_c/k as an expression of the heat-transfer parameter, where k_c is the effective conductivity of heat and k is the thermal conductivity of fluid. And they used the gap width δ or the outer diameter of the inner cylinder D_1 as a characteristic length of the Grashof number Gr same as Beckmann [1] or Kraussold [2] did. The coordinate system for this problem is shown in Fig. 1. Grigull *et al.* [4] used the gap width δ as a characteristic length of the mean

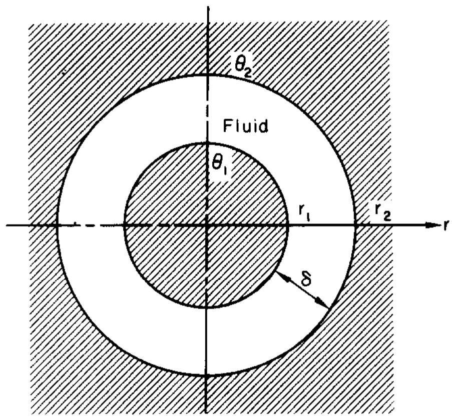

FIG. 1. Horizontal cylindrical annulus and co-ordinate system.

Nusselt number $\overline{Nu}$ and also of the Grashof number Gr_δ. Lis [6] used k_c/k as an expression of the heat-transfer parameter, and showed experimentally that k_c/k is a function of $X = Ra_{D_1}(1 - D_1/D_2)^{6\cdot 5}$, where Ra_{D_1} is the Rayleigh number with D_1 as a characteristic length.

Thus, each investigator correlates his results in a different way, but none of them is a perfect expression which has a physical meaning. The authors of this paper intend to define clearly the Nusselt number and to propose a new characteristic length of the Grashof number.

THEORY

1. *Nusselt number*

From the definition of the effective conductivity of heat k_c [1, 6], the ratio k_c/k represents the ratio Q_{conv}/Q_{cond}. As in the case of natural convection in a rectangular enclosure, it is reasonable to define the mean Nusselt number $\overline{Nu}$ as the ratio Q_{conv}/Q_{cond}. Defining the mean Nusselt number $\overline{Nu}$ as above, the next relationship is obtained;

$$\frac{k_c}{k} = \frac{Q_{conv}}{Q_{cond}} = \overline{Nu} = \frac{\text{(mean heat-transfer coef.)} \times \text{(characteristic length)}}{\text{(thermal conductivity)}} \quad (1)$$

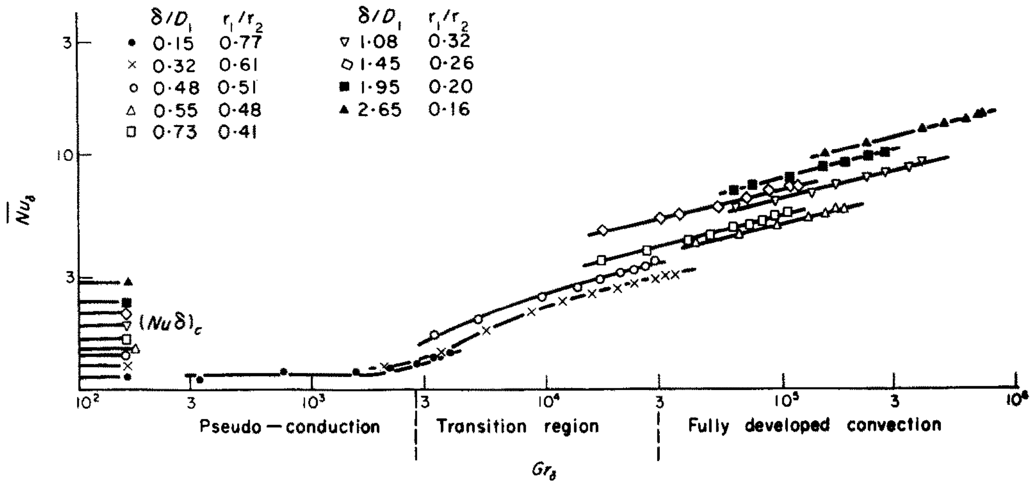

FIG. 2. Correlation of heat-transfer coefficients by Gribull *et al.* [4].

The characteristic length in the Nusselt number depends on the radius where the heat-transfer coefficient is calculated. Suppose that the mean heat-transfer coefficient is calculated at the inner radius of the outer cylinder r_2, the heat-transfer rate by convection becomes

$$Q_{conv} = \bar{h}_2 . 2\pi r_2(\theta_1 - \theta_2) \quad (2)$$

where $\bar{h}_2$ is the mean heat-transfer coefficient at r_2. The heat-transfer rate by pure conduction Q_{cond} becomes

$$Q_{cond} = \frac{2\pi k(\theta_1 - \theta_2)}{\ln(r_2/r_1)}. \quad (3)$$

Substituting equations (2) and (3) in equation (1), the mean Nusselt number $\overline{Nu}$ obtained is

$$\overline{Nu} = \frac{\bar{h}_2 . [r_2 \ln(r_2/r_1)]}{k} \quad (4)$$

As mentioned above, the mean Nusselt number calculated at the outer radius of the inner cylinder r_1 becomes

$$\overline{Nu} = \frac{\bar{h}_1 . [r_1 \ln(r_2/r_1)]}{k} \quad (5)$$

and is found to be

$$\overline{Nu} = \frac{\bar{h}_2 . [r_2 \ln(r_2/r_1)]}{k} = \frac{\bar{h}_1 . [r_1 \ln(r_2/r_1)]}{k} = \frac{Q_{conv}}{Q_{cond}}. \quad (6)$$

In the region of conduction only, the mean Nusselt number defined above becomes unity. On the other hand, the local Nusselt number at r_2 can be defined as

$$Nu_{r_2} = \frac{h_2 . [r_2 \ln(r_2/r_1)]}{k}, \quad (7)$$

and also the local Nusselt number at r_1 becomes

$$Nu_{r_1} = \frac{h_1 . [r_1 \ln(r_2/r_1)]}{k}. \quad (8)$$

Thus the characteristic length in the local Nusselt number depends on the radius where the heat-transfer coefficient is calculated.

2. *Grashof number*

The reason that many of previous investigators selected the gap width δ as a characteristic length of the Grashof number is thought to be related with the forced convection heat transfer in annuli. In such a case, the characteristic length is generally defined as the hydraulic diameter d_e which is the ratio of four times the sectional area ($4S$) to the wetted perimeter l as follows,

$$d_e = \frac{4S}{l} = \frac{4\pi(r_2^2 - r_1^2)}{2\pi(r_2 + r_1)} = 2\delta. \quad (9)$$

$$\therefore r_e = \delta. \tag{10}$$

However, it is not adequate to use δ as a characteristic length of natural convection in horizontal cylindrical annuli, because the flow pattern in annuli is completely different. Therefore the authors intend to propose another characteristic length. It is stated that the temperature of a fluid reaches an average temperature $\theta_m = (\theta_1 + \theta_2)/2$ at the radius r_m in the case of pure heat conduction. From equation (3), it becomes

$$Q_{\text{cond}} = \frac{2\pi k(\theta_1 - \theta_2)}{\ln(r_2/r_1)} = \frac{2\pi k(\theta_1 - \theta_m)}{\ln(r_m/r_1)}, \tag{11}$$

$$r_m = \sqrt{(r_1 r_2)}, \tag{12}$$

that is, the radius r_m is the geometric mean of r_1 and r_2. It is well known from previous investigations that the heat-transfer coefficient by natural convection increases with an increase in the gap width δ, even if the annuli have the same r_m. In order to include the effect of the gap width, the authors selected the combined form of r_m and $\ln(r_2/r_1)$ as a characteristic length of the Grashof number as follows,

$$r_m \,.\, \ln(r_2/r_1) = (\sqrt{r_1 r_2}) \,.\, \ln(r_2/r_1). \tag{13}$$

It seems reasonable to select this group, because in the limiting case for $r_1 \approx r_2$, the characteristic length $r_m \,.\, \ln(r_2/r_1)$ approaches δ which is adequate as a characteristic length for heat transfer between two parallel plates with a gap width δ. Consequently the Grashof number is defined as follows,

$$Gr_m = \frac{g\beta(\theta_1 - \theta_2)\,[r_m \ln(r_2/r_1)]^3}{\nu^2}. \tag{14}$$

RESULTS

As mentioned in the introduction, Grigull *et al.* [4] correlated their experimental data by using $\overline{Nu_\delta}$ and Gr_δ as shown in a table in their paper. Nu_δ and Gr_δ can be converted to the $\overline{Nu}$ and Gr_m defined in this paper using the following equations

$$\overline{Nu} = \frac{n}{\delta}\ln(r_2/r_1) \,.\, \overline{Nu_\delta} \tag{15}$$

$$Gr_m = \left(\frac{r_m}{\delta}\ln(r_2/r_1)\right)^3 . \, Gr_\delta. \tag{16}$$

The rearrangement of their data using $\overline{Nu}$ and Gr_m is shown in Fig. 3. Their data are fully correlated in a single straight line. An experimental equation is obtained as follows,

$$\overline{Nu} = 0{\cdot}18\, Gr_m^{\frac{1}{4}} \qquad (Pr = 0{\cdot}71, \quad Gr_m \geq 10^4). \tag{17}$$

On the other hand Beckmann [1] correlated his experimental data for using $\overline{Nu}$ and Gr_{D_1} which is the Grashof number based on D_1 as a characteristic length. Gr_{D_1} can be converted to Gr_m as follows:

$$Gr_m = \frac{1}{8}\left\{\left(\sqrt{\frac{\gamma_2}{\gamma_1}}\right)\ln\left(\frac{r_2}{r_1}\right)\right\}^3 . \, Gr_{D_1}. \tag{18}$$

Rearrangement of his data using $\overline{Nu}$ and Gr_m is shown in Fig. 4. His data also agree well with equation (17).

The authors intend to use the Rayleigh number in order to correlate the data for the fluids other than air. The Rayleigh number Ra_m is defined as follows.

$$Ra_m = Pr \,.\, Gr_m. \tag{19}$$

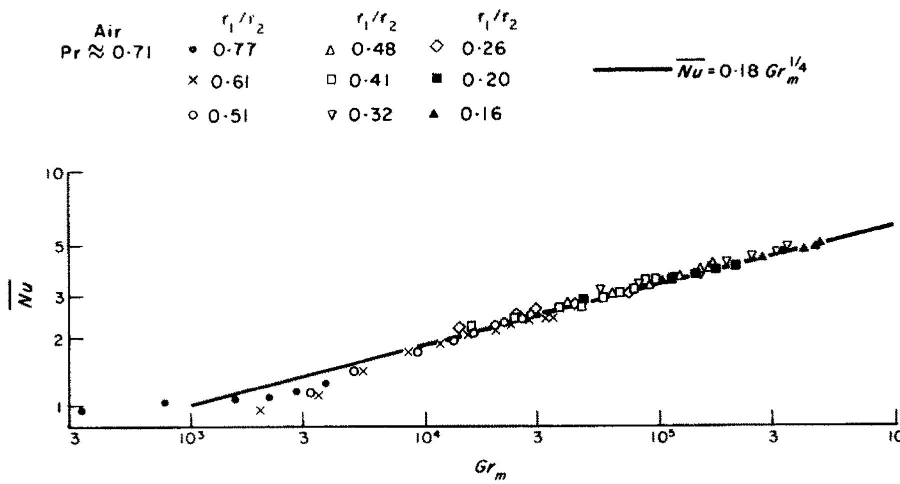

FIG. 3. Rearrangement of data obtained by Grigull *et al.* [4] using $\overline{Nu}$–Gr_m.

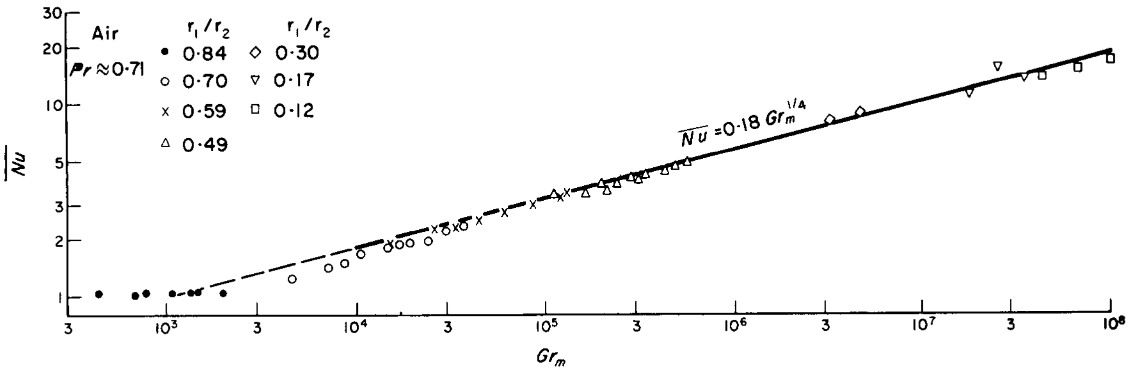

FIG. 4. Rearrangement of data obtained by Beckmann [1] using $\overline{Nu}$–Gr_m.

Converting Gr_m of the equation (17) to Ra_m using $Pr = 0{\cdot}71$, the following equation is obtained.

$$\overline{Nu} = 0{\cdot}20\, Ra_m^{\frac{1}{4}} \qquad (Ra_m \geqslant 7{\cdot}1 \times 10^3). \qquad (20)$$

Kraussold [2] made the experiments using water and transformer oil. Rearrangement of his data using $\overline{Nu}$ and Ra_m is shown in Fig. 5. As the Prandtl number of water or transformer oil significantly changes according to the temperature, the coincidence of the data is not so good as that for air. But his experimental data almost agree with the equation (20).

CONCLUSIONS

The authors propose a new method of correlating the heat-transfer coefficients for natural convection in horizontal cylindrical annuli· The heat-transfer coefficients are well correlated by the mean Nusselt number $\overline{Nu}$ and the Grashof number Gr_m defined as follows,

$$\overline{Nu} = \frac{Q_{\text{conv}}}{Q_{\text{cond}}} = \frac{\bar{h}_1 \,.\, [r_1 \ln (r_2/r_1)]}{k} = \frac{\bar{h}_2 \,.\, [r_2 \ln (r_2/r_1)]}{k}.$$

$$Gr_m = \frac{g\beta(\theta_1 - \theta_2)\,[(\sqrt{r_1 r_2}) \ln (r_2/r_1)]^3}{\nu^2}.$$

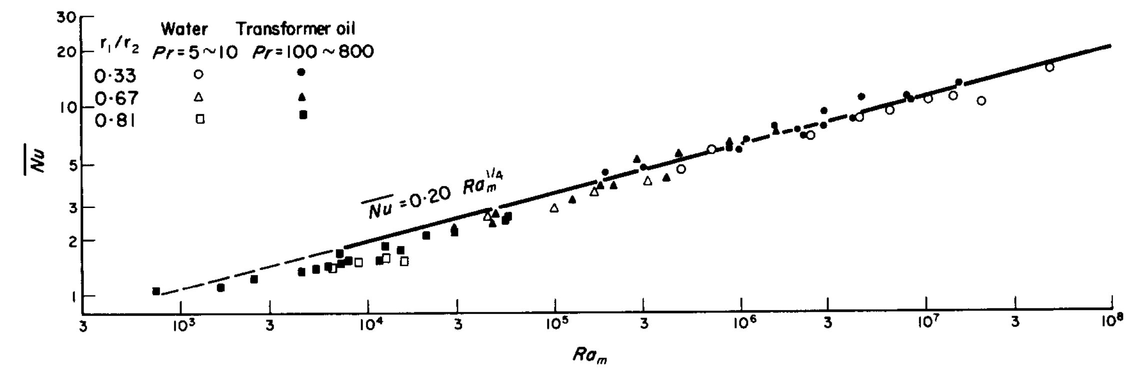

FIG. 5. Rearrangement of data obtained by Kraussold [2] using $\overline{Nu}$–Ra_m.

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DISPLACEMENT THICKNESS OF AN UNSTEADY BOUNDARY LAYER WITH SURFACE MASS TRANSFER

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NOMENCLATURE

a_∞, ambient sound speed;
F, function defining the displacement surface;
h, a normal distance from the plate, slightly larger than the boundary-layer thickness;
$M_\infty(t)$, instantaneous plate Mach number, $U_\infty(t)/a_\infty$;
$\dot{m}(x, t)$, surface mass flux normal to the plate, equal to $\rho_w v_w$;
$Re(x, t)$, Reynolds number, $xU_\infty(t)/\nu$;
t, time;
(x, y), Cartesian coordinate system, fixed relative to the plate, see Fig. 1;
(u, v), velocity vector of the boundary layer flow field;
$U_\infty(t)$, plate speed;
α, a constant of order unity;
$\delta^*(x, t)$, a quantity defined by equation (5.1);
$\delta_\rho(x, t)$, a quantity defined by equation (5.2);
$\Delta^*(x, t)$, displacement thickness for unsteady flows with surface mass transfer;
ν, "scaled" kinematic viscosity, being a constant equal to $C\nu_\infty$;
$\rho(x, t)$, density.

Subscripts
e (or ∞), conditions at the outer edge of the boundary layer;
w, conditions at the surface uf the plate;
1, 2, conditions at $x = x_1$ and $x = x_2$, respectively.

1. INTRODUCTION

THE CONCEPT of the displacement thickness of a viscous boundary layer is very useful and important, particularly in studying the viscous-inviscid interaction effects [1]. For steady flows with no surface mass transfer, the procedure for calculating this thickness is standard and straightforward (see e.g. Schlichting [2]). When the boundary layer is unsteady, the displacement surface can also be found by regarding such a surface as a fictitious solid boundary (impermeable) placed in the given free stream, and the unsteady, inviscid boundary condition on such a boundary leads to a normal velocity distribution just the same as that given by the boundary layer solutions at the outer edge. This was first done by Moore and Ostrach [3] who derived a differential equation for such a surface, valid for general, unsteady boundary layers, but without surface mass transfer.

With surface mass flux, the effective displacement thickness of a boundary layer has been studied by Mann [4] for the simple geometry of a flat plate in parallel motion. The analysis was later generalized by Hayasi [5] to account for arbitrary geometries. However these analyses were all aimed at steady flow situations.

In many practical applications, such as flights of rockets, missiles or re-entry vehicles, a continuously varying flight speed is often encountered. It is therefore of importance to

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